Pollution Control and the Ramsey Problem

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Abstract. Pollution is an inevitable by-product of production and is only gradually dissolved by the environment. It can be reduced by producing less and by cleaning up the environment, but neither occur when they are left to the market. Cleaning activities and the optimal emission charges increase with the stock of pollutants. When one allows for pollution of the environment in the classical Ramsey problem, the capital stock is less than in the market outcome and *a fortiori* less than under the golden rule. The analysis distinguishes between stock and flow externalities arising from pollution. An increase in impatience can lead to more capital accumulation, even though this leaves less room for current consumption.

Key words. Pollution control. abatement activities, capital accumulation, Ramsey model.

1. Introduction

Pollution is an inevitable by-product of production, which damages the environment and is only gradually dissolved. Since there are no private markets for pollution rights, market outcomes are inefficient and give rise to too much production and pollution (e.g., Dasgupta, 1982). The four standard approaches to environmental policy are property rights, binding quota restrictions, Pigouvian taxes and subsidies, and markets for pollution permits. The problem with property rights is that they are difficult to define, since many pollution and environmental problems are characterised by the problem of the common. The problem with quotas is that emission standards are difficult to enforce and lead to high administrative costs, whilst the difficulty with emission charges and permits is that they only apply in well-behaved situations without non-convexities. The present paper focuses, nevertheless, on optimal emission charges in the context of a dynamic pollution problem embedded in the Ramsey model. Since pollution is essentially a problem of missing markets, the emission charges correspond to the social price of an additional unit of pollution.

Section 2 considers pollution control when waste is a by-product of production. Distinction is made between the social costs of both the flow and the stock of pollutants. The market outcome and the socially optimal outcome are compared. Because in the absence of government intervention there will be too much pollution and not enough abatement activities,

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Pigouvian tax and subsidy schemes can be used to sustain the socially optimal outcome in a decentralised market economy. Section 3 reviews the Ramsey problem of optimal consumption and capital accumulation and thereby sets the scene for later sections. Section 4 discusses how environmental stock and flow externalities can be added to the Ramsey framework and discusses previous work in this area. Sections 5 and 6 focus on Pigouvian tax and subsidy schemes and show how they can be used to correct for environmental flow and stock externalities in the Ramsey problem. Section 7 discusses the socially optimal level of abatement activities within the context of a Ramsey model with social costs associated with a stock of waste products. It is then possible that more impatience leads to more capital accumulation, despite the fact that this leaves less room for current private consumption. Section 8 analyses why the market invests too little in clean technology and Section 9 briefly considers the implications for environmental policy when renewable resources are used as factors of production. Section 10 concludes the paper.

2. Pollution Control

It is assumed for the time being, that there is no investment in physical capital; later sections relax this assumption. Consumption, C, is thus production of goods, Y, minus the amount of output that is used to clean up the environment, A. Production is limited by the availability of given factors of production, so $Y \leqslant Y_{max}$. Net social benefits of consumption are given by $B(C)$, $B' > 0$, and marginal benefits are decreasing in the level of consumption, $B'' < 0$. One reason may be that as a society consumes more, it needs to produce more and thus needs to work harder and forego more leisure in order to secure an additional unit of consumption. To rule out non-positive levels of consumption, it is assumed that $B'(0) = \infty$. Pollution is an inevitable by-product of production, αY, where $\alpha > 0$ denotes the emission-output ratio. The emission-output ratio can be improved by investment in new technology, but this will be ignored for the time being (see Section 8). The stock of pollutants, S, follows from

$$\dot{S} = \alpha Y - \sigma(A)S, \quad S(0) = S_0, \quad \text{given}, \tag{2.1}$$

where $\sigma(A) \geqslant 0$ denotes the rate at which pollutants are dissolved by the environment. There is an amount A of total production Y devoted to cleaning-up activities. The rate at which pollutants are dissolved is higher when more efforts are made to clean up the environment, $\sigma' > 0$. Returns to such efforts are diminishing, $\sigma'' < 0$. Pollutants such as DDT dissolve very slowly, whilst herbicides dissolve quite quickly.

The social welfare function is given by

$$W \equiv \int_0^\infty \exp(-\theta t)\,[B(Y-A) - D_F(\alpha Y) - D_S(S)]\,\mathrm{d}t, \tag{2.2}$$

where $\theta > 0$ denotes the social rate of discount, $D_F(\alpha Y)$ (with $D'_F > 0$ for $\alpha Y \neq 0$ and $D'_F(0) = 0$) denotes the social damage caused by the flow of pollution (e.g. due to noise) and $D_S(S)$ with ($D'_S > 0$ for $S \neq 0$ and $D'_S(0) = 0$) denotes the social damage caused by the stock of pollutants (e.g. due to the stock of SO_2 in air). Pollutants often display threshold effects; for example, below a certain level of smog trees survive, but above this level trees do not survive. This leads to non-convexities and means that it is difficult to rely on Pigouvian taxes and subsidies in the design of environmental policy. Here it is assumed that the marginal damage increases with the stock of pollutants, so that the damage functions are convex, $D''_F, D''_S \geqslant 0$.

The following conditions must be satisfied for optimal social welfare:

$$B'(Y-A) = \alpha[(D'_F(\alpha Y) + \tau] = \sigma'(A)S\tau \tag{2.3}$$

$$D'_S(S) - \sigma(A)\tau + \dot{\tau} = \theta\tau \tag{2.4}$$

where τ denotes the optimal emission charge per unit of pollution, αY. The second equality of (2.3) holds only if $A > 0$; if $D_S(.) = 0$ then it is optimal to have $A = \tau = 0$ and the second equality of (2.3) is replaced by a greater than inequality. The shadow price (co-state) associated with the stock of pollutants corresponds to $-\tau$, because the concentration level of pollutants is a stock with a negative social value. Equation (2.3) demands that the marginal social benefits of consumption should equal the marginal flow plus stock damage to the environment due to consumption.

In the absence of any stock damage, $D_S \equiv 0$, emission charges are zero, $\tau = 0$, so that the marginal social benefits must equal marginal social flow damage, $B'(C) = \alpha D'_F(\alpha Y)$ giving rise to $C = C^S$. The market outcome, however, corresponds to a maximum level of production and consumption, $C^M = Y^M = Y_{\max}$. Since the market does not internalise the externality to consumers associated with pollution arising from production, the levels of production and consumption are too high (see Figure 1). The socially optimal outcome can be sustained by levying a consumption tax at a rate equal to $\alpha D'_F(\alpha C^S)$ and redistributing the revenues in a lump-sum fashion. Alternatively, it can be sustained by levying a pollution tax equal to $D'_F(\alpha C^S)$ per unit of emitted pollutants.

Now consider stock externalities and, for the time being, assume that the rate at which pollutants are dissolved is constant (A = constant). In that case equation (2.3) can be solved to give

$$Y = \bar{Y}(A, \tau) \tag{2.5}$$

where $\bar{Y}_A = B''/(B'' - \alpha^2 D''_F) > 0$ and $\bar{Y}_\tau = \alpha/(B'' - \alpha^2 D''_F) < 0$. Hence, the optimal level of production and thus of pollution increases when efforts are made to clean up the environment and when the social cost of the stock of pollutants decreases, for example as a consequence of less concern with stock damage.

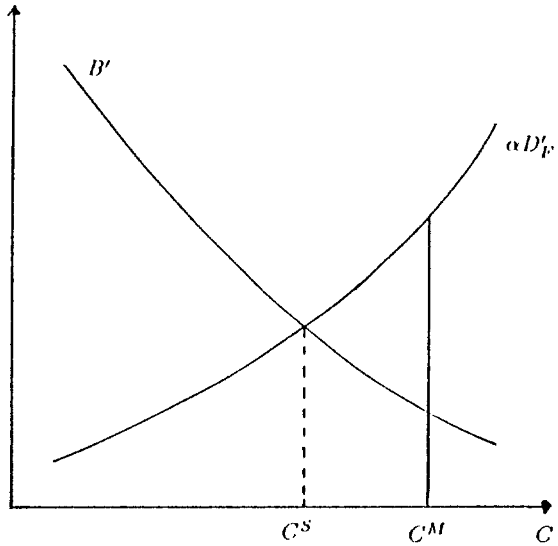

Fig. 1. Market and social outcomes in the presence of flow externalities only.

Alternatively, when efforts to clean up the environment are endogenous, equation (2.3) can be solved to give

$$A = A(\tau, S), \quad Y = Y(\tau, S) \tag{2.6}$$

where $A_\tau = (B''\overline{Y}_\tau - \sigma'S)/[\sigma''S\tau + B''(1 - \overline{Y}_A)]$, $A_S = -\sigma'\tau/[\sigma''S\tau + B''(1 - \overline{Y}_A)] > 0$, $Y_\tau = \overline{Y}_A A_\tau + \overline{Y}_\tau$ and $Y_S = \overline{Y}_A A_S > 0$. When the stock of pollutants increases, the efforts to clean up the environment increase and ceteris paribus one can afford to have a higher level of production. The effects of a decrease in the social cost of the stock of pollutants on the level of production and efforts to clean up the environment are ambiguous: the direct effect is for the level of production to increase, the marginal benefit of consumption to fall and thus the level of cleaning-up activities to increase, but the indirect effect is to reduce the social need for cleaning-up activities so that these efforts are reduced. Which way it goes is part of the old-age dispute in environmental discussions! The optimists say production must go up in order to be able to afford to clean up the environment, whilst the pessimists argue that production must go down, *even* if this leaves less scope for cleaning-up activities, as pollution as a by-product of production dominates all else. This dispute can only be settled by empirical evidence. If there are no flow externalities ($D_F \equiv 0$) present, then $\alpha = \sigma'S$, $\overline{Y}_A = 1$, $Y_\tau = \overline{Y}_\tau = \alpha/B'' < 0$, $A_\tau = 0$ and $A_S = -\sigma'/\sigma''S$. An increase in the social cost of pollutants does not affect cleaning-up activities, $A_\tau = 0$, because the emission-output ratio (α) equals the marginal increase in depreciation arising from cleaning-up activities. Production Y does decrease, $Y_\tau < 0$, giving support to the camp of pessimists.

Equation (2.4) says that in equilibrium the social rate of return on the stock of pollutants, i.e., the marginal social damage minus the rate of depreciation plus the expected capital losses, should equal the social rate of discount. This, of course, resembles the arbitrage equation familiar from, for example, Hotelling's theory of exhaustible resource depletion.

The dynamics of the stock of pollutants and the optimal emission charge are given by:

$$\dot{S} = \alpha Y(\tau, S) - \sigma(A(\tau, S))S, \quad S(0) = S_0 \tag{2.7}$$

$$\dot{\tau} = [\theta + \sigma(A(\tau, S))]\tau - D'_S(S). \tag{2.8}$$

The phase diagram for the case that cleaning-up activities and thus depreciation are exogenous is presented in Figure 2. The $\dot{S} = 0$-locus slopes downwards, because a higher stock of pollutants must be caused by a higher level of production induced by a lower level of the optimal emission charge. The $\dot{\tau} = 0$-locus slopes upwards, because an increase in the stock of pollutants increases the marginal damage caused to the environment and thus requires a higher emission charge. The equilibrium is at the intersection of these two loci,[1] whilst the transient behaviour is described by saddlepoint dynamics. A fall in the emission-output ratio shifts the $\dot{S} = 0$-locus down and thus moves the equilibrium from E to E'. On impact the optimal emission charge and level of production drop immediately (move from E to A) and subsequently

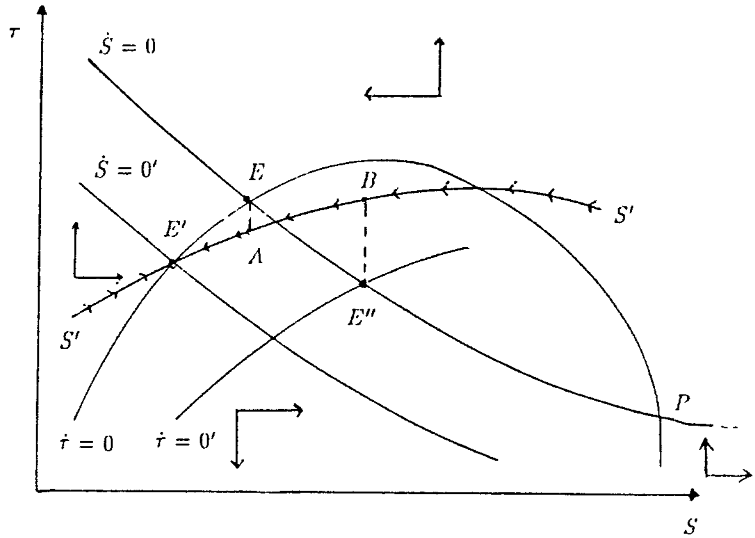

Fig. 2. Emission charges and stock of pollutants in the presence of stock externalities ($\sigma(A) \equiv \sigma$).

they are further reduced until the stock of pollutants has fallen to its new equilibrium value (move from A to E' along the new saddlepath $S'S'$). On the other hand, an increase in the social rate of discount (or the government's pure rate of time preference) shifts the $\dot{\tau} = 0$-locus down and thus moves the equilibrium from E to E''. As a result of this increase in impatience, the stock of pollutants increases and the optimal emission charge falls. A subsequent exogenous increase in efforts to clean up the environment shifts the equilibrium from E'' to E'. The impact effect corresponds to a move from E'' to B, whilst the transient effect corresponds to a move from B to E'.

An important issue is what are the appropriate Pigouvian taxes to sustain socially optimal behaviour as a decentralised market outcome. Since there are two externalities (flow and stock), it is sensible to have two taxes: one on gross emissions (τ_G) as they affect the flow and one on net emissions (τ_N) as they affect the stock of pollutants. In a decentralised market economy firms maximise (taking τ_G, τ_N and S as given) $B(Y - A) - \tau_G \alpha Y - \tau_N[\alpha Y - \sigma(A)S]$, which gives the social optimum provided $\tau_G = D'_F(\alpha Y)$ and $\tau_N = \tau$. Alternatively, gross emissions are taxed at the rate $\tau_G + \tau_N$, and cleaning the environment is subsidised at the rate τ_N. Provided the activity of cleaning up the environment (A) can be decentralised, only two Pigouvian taxes are needed to sustain the first-best outcome. Any excess revenues should be redistributed in a lump-sum fashion to the private sector.

3. The Ramsey Problem

The following sections discuss Ramsey problems with pollution. In order to have a benchmark, this section first gives a bird eye's view of the Ramsey problem. A more detailed treatment may be found in Blanchard and Fischer (1989).

Ramsey's model is the classic framework for studying the optimal intertemporal allocation of resources. Preferences of the infinitely-lived representative family (or dynasty) are equal to

$$W \equiv \int_0^\infty \exp[-\theta t] B(C(t))\,\mathrm{d}t \tag{3.1}$$

where θ is a constant[2] and $C(t)$ denotes per-capita private consumption at time t. Production is characterised by constant returns to scale. Per-capita output, Y, is given by the intensive-form production function, $f(K)$, $f' > 0$, $f'' < 0$, $f(0) = 0$, $f'(0) = \infty$, $f'(\infty) = 0$ where K denotes the per-capita capital stock. Since output is either consumed or invested, one has

$$\dot{K} = f(K) - C - (\delta + n)K, \quad K(0) = K_0, \tag{3.2}$$

where n denotes the exogenous growth rate of labour supply and δ denotes the rate of physical depreciation of the capital stock. Alternatively, income is either saved (in the only asset that is available in this economy) or consumed. Because there are no externalities whatsoever, the fundamental theorem of welfare economics says that the market outcome is socially optimal and thus corresponds to the command optimum. The social planner maximises (3.1) subject to (3.2). This yields the famous Keynes—Ramsey rule

$$\dot{C} = [f'(K) - \theta - n - \delta]\eta(C)C, \tag{3.3}$$

where $\eta \equiv -B'/(CB'') > 0$ denotes the instantaneous elasticity of intertemporal substitution, and the transversality condition

$$\lim_{t \to \infty} [K(t)B'(C(t))\exp(-\theta t)] = 0. \tag{3.4}$$

Equation (3.3) is nothing more than the continuous-time efficiency statement that the marginal rate of substitution between consumption at two points of time should equal the marginal rate of transformation. In a market economy firms maximise profits and set the marginal product of capital to the user cost of capital, i.e., $f'(K) = r + \delta$ where r denotes the market rate of interest. Similarly, families maximise the discounted value of life-time consumption, (3.1), subject to their intertemporal budget constraint, which yields the "tilt" $\dot{C} = (r - n - \theta)\eta C$ and consumption as the propensity to consume times the sum of human wealth plus non-human wealth. Hence, when the market rate of interest, r, exceeds the pure rate of time preference, $\theta + n$, society prefers to save and defer consumption and thus consumption increases over time. Conversely, if $r < \theta + n$, consumption falls over time. The higher the elasticity of substitution, η, the easier it is to substitute present consumption for future consumption and vice versa. For the case of a constant coefficient of relative risk aversion, β, one has $B(C) = C^{1-\beta}/(1 - \beta)$, $\beta > 0$, $\beta \neq 1$, $B(C) = \log(C)$, $\beta = 1$, and then $\eta \equiv (1/\beta)$ corresponds to the instantaneous elasticity of intertemporal substitution.

A substantial literature exists on golden rules of economic growth (e.g. Phelps, 1966). The condition that maximises the long-run level of per-capita consumption is called the golden rule. This is $f'(K) = n + \delta$ and yields $K^G = \phi(n + \delta)$, $\phi' < 0$. The condition that ensures a steady per-capita rate of consumption is called the modified golden rule. It is given by $f'(K) = n + \delta + \theta$ and yields $K^{MG} = \phi(n + \delta + \theta) < K^G$, so that the capital stock is reduced below the golden rule level due to the impatience of society. The steady-state market real rate of interest, $r(\infty) = n + \theta$, is thus determined by tastes alone, whilst technology then determines the steady-state stock of capital, output and consumption. The phase diagram associated with equations (3.2)—(3.3) is presented in Figure 3.

The equilibrium E is the only steady state that satisfies the optimality conditions and that is also saddlepoint stable.

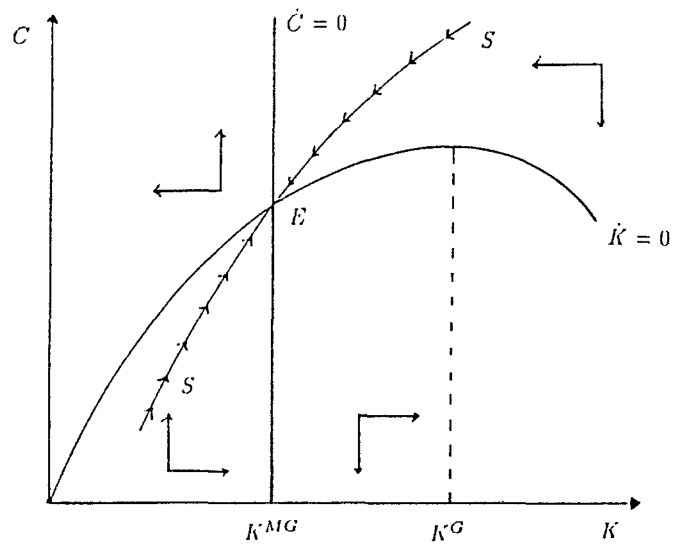

Fig. 3. The modified Golden Rule.

4. Environment and the Ramsey Problem

Here capital is incorporated into the model of Section 2 or, alternatively, environmental issues are incorporated into the model of Section 3. The social welfare function is given by

$$W = \int_0^\infty \exp(-\theta t)\,[B(C) - D_F(af(K)) - D_S(S)]\,\mathrm{d}t. \tag{4.1}$$

The government maximises this welfare function subject to

$$\dot{K} = f(K) - \delta K - C - A, \quad K(0) = K_0 \quad \text{given,} \tag{4.2}$$

$$\dot{S} = af(K) - \sigma(A)S, \quad S(0) = S_0 \quad \text{given.} \tag{4.3}$$

For simplicity, it has been assumed that there is no population growth (n = 0). This is a control problem with two state variables. It is well known that such problems are hard to solve analytically. However, due to the special character of this particular problem, a good deal can be said about the solution. Before proceeding, it is worthwhile to compare the present approach with previous approaches which deal with capital accumulation and the environment simultaneously. The first reference is probably Keeler, Spence, and Zeckhauser (1971) who use a model where output depends through a neoclassical production function on the capital stock and is allocated to either consumption, investment or reduction of the emission-output ratio. Only the stock of pollutants affects welfare in this classical study. The control variables are the proportions of output devoted to consumption and clean technology. A unique steady state is shown to exist with either no attempt to lower emission-output ratio (Modified Golden Age) or with a positive share of output devoted to cleaner technology (Murky Age). This study does not discuss abatement activities. Forster (1973) and Gruver (1976) also look at pollution and optimal capital accumulation, but they only consider the effects of the flow of pollution on welfare. An important text is Mäler (1974); Chapter 3 is particularly relevant. The major differences with the present model are that Mäler does not allow for abatement activities, that pollution is caused by consumption rather than by production and that total production over time is bounded by the limited availability of an exhaustible resource. Becker (1982) works along the lines of Brock's (1977) model of capital accumulation and environmental quality. He does not allow for abatement activities either. In his model there is joint production, so that output of goods is given by the production frontier $Y = F(K, Z)$, F_K, $F_Z > 0$, where Z denotes the waste generated by production. This allows for substitution possibilities between the capital stock and the emission of rubbish when one produces a given amount of output. As a consequence the development of the stock of waste products, S, is given by $\dot{S} = Z - \sigma S$. Finally, Becker uses the maximin criterion for social welfare. He shows that an increase in the rate of time preference can lead to a higher steady-state capital stock and a lower level of pollution, even though this would leave less room for private consumption. This result sounds counter-intuitive, because normally one expects that less concern for future generations leads to more pollution. The difference with a recent paper by Musu (1989) lies in the fact that in Musu's model capital can be used for production and abatement and can instantaneously be transferred from one activity to another. None of these authors take account of a direct effect of the flow of pollution on welfare. All of them establish the existence of a steady state which is shown to be locally asymptotically stable and of which the comparative statics are analysed. The work of Tahvonen (1990) should be mentioned here as well, be it that he pays almost exclusive attention to the harvesting of a renewable resource in connection with pollution control. Tahvonen and Kuuluvainen (1990) provide a very interesting analysis of a model with accumulation of capital and of waste products very similar to the present one. The main difference is that attention is focused on substitution possibilities between capital and the generation of waste products when producing a given amount of output, although the use of resources for abatement activities is ignored.

In order to keep the analysis manageable, a distinction is made here between the case where there is only flow damage and the case where there is only stock damage arising from pollution.

5. Flow Externalities and the Ramsey Problem

This section assumes that there is no detrimental effect of the stock of pollutants on welfare ($D_S(S) \equiv 0$). Then, obviously, there is no case for pollution abatement, so that $A = 0$. Since there are no social costs associated with the stock of pollutants, the problem of maximising (4.1) subject to equations (4.2)—(4.3) is relatively easy to solve.

Individual households are too small to care about pollution as a by-product of production but the social planner does internalise such flow externalities. It is clear that the fundamental theorem of welfare economics no longer holds. The best line of attack seems to characterise the socially optimal outcome, contrast it with the market outcome discussed in Section 3, and show what kind of scheme of Pigouvian taxes and subsidies ensures that the market produces a socially optimal outcome.

The transversality condition (3.4) is unaffected, but the Keynes—Ramsey rule is modified as follows:

$$\dot{C} = \{f'(K)\,[1 - \alpha D'_F(\alpha f(K))/B'(C)] - \theta - \delta\}\,\eta(C)C. \qquad (5.1)$$

Figure 4 shows how the modified golden rule is affected by flow externalities arising from pollution. The $\dot{C} = 0$-locus is given by

$$B'(C) = \left(\frac{\alpha D'_F(\alpha f(K))f'(K)}{f'(K) - \delta - \theta}\right) \qquad (5.2)$$

and requires that the marginal social benefit of an additional unit of private consumption equals the marginal damage to the environment arising from the additional production required to satisfy consumption. The $\dot{C} = 0$-locus can easily be shown to be downward-sloping and to go through the point $C = 0$, $K = K^{MG}$. Hence, the phase diagram is as in Figure 4, rather than Figure 3. It is easy to see that the steady-state stock of capital is smaller than the one under the modified golden rule, because $\alpha D'_F/B' > 0$ and thus from equation (5.2) $r > \theta$. The steady state is given by point E'.

It is also instructive to consider the $\dot{C} = 0$-locus in $r - K$ space. It is given by

$$r = R(K) \equiv \left(\frac{\delta\alpha D'_F(\alpha f(K)) + \theta B'(f(K) - \delta K)}{B'(f(K) - \delta K) - \alpha D'_F(\alpha f(K))}\right). \qquad (5.3)$$

This locus starts at $R(0) = \theta$ and can easily be shown to be upward-sloping. The $\dot{K} = 0$-locus in $r - K$ space is, of course, $f'(K) = r + \delta$, and is downward-sloping. Figure 5 illustrates how the intersection of these two loci gives the equilibrium levels of the interest rate and the capital stock, say, r^P and K^P. The presence of social costs arising from a flow of pollutants means that the steady-state capital stock is below and the steady-state real interest

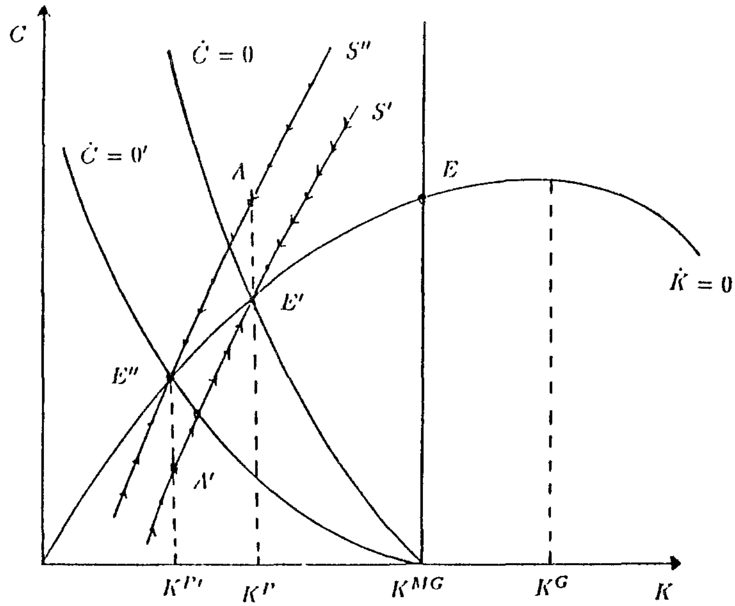

Fig. 4. Flow externalities and the Ramsey problem.

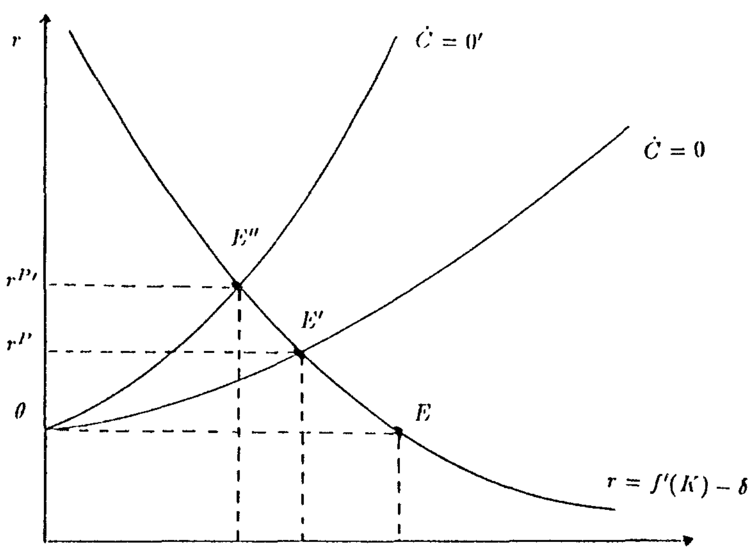

Fig.5. Pollution and the modified golden rule.

rate is above the modified golden rule, given by point E. This gives rise to the "Polluted Golden Age", E'. The strict separation between tastes determining the real interest rate and technology then determining the capital stock and the level of consumption disappears, because technology and pollution externalities now interact to drive up the real interest rate.

An increase in the emission-output ratio, α, steepens the $\dot{C} = 0$-locus in $r - K$ space and increases the steady-state real interest rate and reduces the socially optimal steady-state capital stock (move from E' to E''). Because less capital is being used for environmental reasons, less is produced and thus less is consumed. Figure 4 also shows that, even though the steady-state level of consumption must fall, on impact consumption jumps up (from E' to A) before it declines (move along $S''S''$ from A to E''). This misadjustment of consumption in response to an increase in pollution is the only way in which consumption can be cut in the long run. Similarly, when one manages to cut the emission-output ratio, consumption in the socially optimal outcome must first fall before it can increase in the long run (path $E''A'E'$). This reminds one of the French proverb: "reculer pour mieux sauter".

The comparative statics with respect to the depreciation, δ, and the pure rate of time preference, θ, can be performed in a similar fashion. An increase in either of these two parameters shifts the $\dot{C} = 0$-locus in $r - K$ space up and consequently the equilibrium real rate of interest increases whilst the equilibrium capital stock falls. Hence, the levels of production and consumption fall.

The easiest way to sustain the socially optimal outcome in a decentralised market economy is to levy a Pigouvian tax on pollution $\alpha f(K)$, at a rate equal to $D'_F(\alpha f(K^S))$ and to redistribute the revenues in a lump-sum fashion to firms.

6. Stock Externalities and the Ramsey Problem

If pollution is detrimental to the welfare of society as a stock, the social planner maximises

$$W = \int_0^\infty \exp[-\theta t]\,[B(C(t)) - D_S(S(t))]\,\mathrm{d}t \tag{6.1}$$

subject to (4.2) and (4.3). The depreciation rate of the stock of pollutants is assumed to be constant, $\sigma(A) = \sigma$, so for the time being there are no resources used to clean up the environment. An interior solution must satisfy the modified Keynes—Ramsey rule

$$\dot{C} = \{f'(K)\,[1 - (\alpha/B'(C))\tau] - \theta - \delta\}\,\eta(C)C \tag{6.2}$$

and the arbitrage rule (cf. (2.8))

$$\dot{\tau} = (\theta + \sigma)\tau - D'_S(S). \tag{6.3}$$

Clearly, the steady-state capital stock is again below the modified golden rule familiar from Ramsey's problem. For a given emission charge, τ, the phase diagram associated with (4.2) and (6.2) is as was presented in Figure 4. The $\dot{C} = 0$-locus is again downward-sloping and intersects the $\dot{K} = 0$-locus below the modified golden rule. An increase in the emission charge, τ, then shifts down the $\dot{C} = 0$-locus and moves the equilibrium from E' to E''. The result is lower levels of steady-state consumption, output and capital. Similarly, the dynamics of S and τ following from (4.3) and (6.3) can be analysed for a given capital stock in the manner discussed in Section 2. The four-dimensional phase dynamics is difficult to handle analytically. However, there should be two eigenvalues with negative real parts associated with the predetermined variables, K and S, and two eigenvalues with positive real parts associated with the non-predetermined variables, C and τ, in order for the system to satisfy the saddlepoint property. This will be the case as long as the rate of time preference, θ, is not too high (see Appendix).

If one assumes that the $S - \tau$ dynamics adjust much more quickly than the $C - K$ dynamics, which is the case when (σ/α) is extremely large, one can solve under the assumption $\dot{\tau} = 0$ and substitute the optimal (steady-state) emission charge

$$\tau = D'_S(\alpha f(K)/\sigma)/(\theta + \sigma) \tag{6.4}$$

directly into the modified Keynes—Ramsey rule:

$$\begin{aligned}\dot{C} = \{&f'(K)\,[1 - (\alpha/B'(C))\,(D'_S(\alpha f(K)/\sigma)/(\theta + \sigma)] - \\ &\theta - \delta\}\,\eta(C)C.\end{aligned} \tag{6.5}$$

The qualitative features of Figure 4 apply, because (6.5) and (5.1) give rise to qualitatively similar $\dot{C} = 0$-loci.

Since attention is restricted to the case where $\sigma(A) = \sigma$, a constant, standard methods yield the following comparative statics results for the full four-dimensional system (see Appendix). A higher emission-output ratio (α) gives again lower steady-state levels for the stock of capital, production, and consumption. The effects of an increase in the abatement coefficient σ are opposite to the effects of a higher α, so it is optimal from society's perspective to have an increase in capital, production and consumption.

Performing the comparative statics around the steady state with respect to θ and α yields

$$\left\{(\alpha\tau - B')f'' + \frac{D''_S\alpha^2(f')^2}{\sigma(\theta + \sigma)} + r(\theta - r)B''\right\}\mathrm{d}K$$

$$= -\frac{B'(2\theta + \sigma - r)}{\theta + \sigma}\,\mathrm{d}\theta - \left[\tau f' + \left(\frac{\alpha f f' D''_S}{\sigma(\sigma + \theta)}\right)\right]\mathrm{d}\alpha. \tag{6.6}$$

The term associated with $\mathrm{d}K$ is positive. It follows that $(\partial K/\partial\alpha) < 0$ as long as σ is high enough. Hence, as long as the environment's rate of breaking down the stock of pollutants is high enough, an increase in the emission-output ratio depresses the optimal stock of capital and thus the rate of production. If θ or σ is sufficiently large, as seems extremely reasonable to assume, a higher rate of time preference induces a lower stock of capital and hence lower rates of consumption and pollution.

7. Abatement Activities and the Ramsey Problem

Consider the problem of Section 6 but now also take into account the optimal use of resources to clean up the environment, A. If an interior solution for C and A is assumed, one must have as necessary conditions equations (4.2) and (4.3) and equations

$$B'(C) = \lambda \tag{7.1}$$

$$\tau\sigma'(A)S = \lambda \tag{7.2}$$

$$\dot{\lambda} = (\alpha\tau - \lambda)f'(K) + (\theta + \delta)\lambda \tag{7.3}$$

and

$$\dot{\tau} = (\theta + \sigma(A))\tau - D'_S(S). \tag{7.4}$$

Here τ is again the optimal emission charge and λ is the shadow price of the stock of capital. Due to the concavity-convexity assumptions there exists a unique steady state, characterised by $\dot{K} = \dot{S} = 0$ and hence $\dot{C} = \dot{A} = \dot{\lambda} = \dot{\tau} = 0$. Furthermore, if θ is small enough, the steady state is locally asymptotically stable. This is not straightforward but it follows from the fact that the Jacobian matrix of the linearized system can be written as

$$J = \begin{pmatrix} H_{11} + \theta I & H_{12} \\ H_{21} & H_{22} \end{pmatrix},$$

where all submatrices are 2×2, $H_{11} = -H_{22}^T$, $H_{21} = H_{21}^T$, $H_{12} = H_{12}^T$. Then it follows that if ψ is a characteristic root of J, $-\psi + \lambda$ is a characteristic root as well (see Appendix). So, for θ small enough, there are two eigenvalues with negative real parts and there exists a stable manifold containing the steady state. It can be shown that this manifold is non-degenerate.

It makes therefore sense to study the steady state in some detail. First note that the steady-state stock of capital is again smaller than the modified golden rule stock of capital. This follows immediately from $\dot{\lambda} = 0$ and $\alpha\tau > 0$ in equation (7.3).

Comparative statics calculations are rather tedious and do not lead to unambiguous outcomes (see Appendix). This is in line with earlier findings by Becker and Musu. As an example consider the effect of increasing the rate of time preference θ. In the Ramsey model one has $f'(K) = \theta + \delta$ so that greater impatience leads to a smaller stock of capital. This effect is present in the extended model as well: the economic subjects want to consume as early as possible and therefore accumulate less capital. On the other hand, if it is not very costly to abate pollution, in other words, if the abatement technology displays a high degree of efficiency (σ' is "large"), a low stock of capital leaves less room for cleaning activities because net output is small, whereas such activities may contribute considerably to welfare through the reduction of pollution. It is not clear a priori which effect dominates, so that a higher rate of time preference may lead to more capital accumulation. Becker (1982, p. 183) also finds that greater impatience may increase the steady state stock of capital but this only occurs in an unstable steady state and if the marginal productivity of capital is increasing in the generation of waste products. This phenomenon also occurred in the present model without abatement activities (see Section 6). In the Musu paper a higher stock of capital may be caused by the increase in the value of the marginal productivity.

8. Investment in Clean Technology[3]

It has been assumed in the previous sections that environmental investment is directed only to activities to clean up the environment. Here the possibility of investing in clean technology is studied. It is assumed that the emission-output ratio can be reduced by investment in clean technology, I, that is $\alpha = \alpha(I)$, $\alpha' < 0$. There are diminishing returns to such investment, so $\alpha'' \geqslant 0$. The social planner maximises the social welfare function

$$W = \int_0^\infty \exp(-\theta t)\,[B(Y-I) - D_F(\alpha(I)Y) - D_S(S)]\,\mathrm{d}t \tag{8.1}$$

subject to

$$\dot{S} = \alpha(I)Y - \sigma S \tag{8.2}$$

$$Y \leqslant Y_{\max}. \tag{8.3}$$

Assuming an interior solution for Y, I and C, the necessary conditions are

$$B'(Y-I) = \alpha(I)\,[D'_F(\alpha(I)Y) + \tau] \tag{8.4}$$

$$B'(Y-I) = -\alpha'(I)Y[D'_F(\alpha(I)Y) + \tau] \tag{8.5}$$

$$\dot{\tau} = (\theta + \sigma)\tau - D'_S(S) \tag{8.6}$$

$$\dot{S} = \alpha(I)Y - \sigma S. \tag{8.7}$$

It follows from equations (8.4) and (8.5) that the level of production and the level of investment in clean technology are directly correlated, $Y =$

$-\alpha(I)/\alpha'(I) \equiv Y(I)$. If $\alpha(I)$ has a constant elasticity, say η, then $Y(I) = I/\eta$ and Y and I go up and down together. The locus for which $\dot{\tau} = 0$ is given by $\tau = D'_S/(\theta + \sigma)$ and is upward-sloping in $\tau - S$ space. The locus for which $\dot{S} = 0$ is given by $S = -\alpha^2/\sigma\alpha' = \alpha(I)I/\sigma\eta \equiv S(I)$, where S' is negative (positive) when η is less than (exceeds) utility. The level of investment in clean technology, I, as a function of τ follows from (8.4):

$$\tau = \frac{B'\left(\dfrac{-\alpha(I)}{\alpha'(I)} - I\right)}{\alpha(I)} - D'_F(-\alpha^2(I)/\alpha'(I)) \equiv \tau(I). \tag{8.8}$$

Together with $S = S(I)$, equation (8.8) gives the $\dot{S} = 0$-locus in $\tau - S$ space. The slope of the $\dot{S} = 0$-locus in $\tau - S$ space is ambiguous. Its sign equals the sign of

$$B'' - \alpha^2 D''_F - B'(\alpha')^3/(-2\alpha(\alpha')^2 + \alpha^2\alpha'').$$

Since $\alpha'' > 0$ the third term need not be negative. It is likely, however, that the entire expression is negative. Then there exists a steady state which is stable and comparative statics is warranted. A higher rate of time preference θ causes a lower position of the $\dot{\tau} = 0$ curve and does not affect the $\dot{S} = 0$ locus. As a consequence the steady state level of pollution increases and its shadow price decreases, as would be expected. An interesting question is whether investments I increase as a consequence of a higher stock of pollutants or not. In the steady state $\alpha(I)Y = \sigma S$. So a higher S calls for higher Y and/or lower I. But Y and I are related by $-\alpha'(I)Y = \alpha(I)$, saying that along an optimum the increase in pollution due to a marginal increase in production should be compensated by allocating additional resources to investment. In the steady state then investment in clean technology, I, increases as the stock of pollutants increases if and only if $-2(\alpha')^2 + \alpha\alpha'' > 0$. So, for example, if the investments in clean technology have linear returns ($\alpha'' = 0$) the $\dot{S} = 0$ locus slopes unambiguously downwards and a higher rate of time preference will decrease the willingness to invest in clean technology. However, suppose that $\alpha(I) = I^{-\eta}(\eta > 0)$. Then $\alpha(I)Y = I^{1-\eta}/\eta$ and in the steady state investment in clean technology increases with the concentration level of pollutants if and only if $\eta > 1$. Hence, if there are large returns to investment in clean technology an increase in impatience can increase the willingness to invest in clean technology.

There are many possible ways of extending the above models to allow for pollution and investment in clean technology. An obvious extension is to embed the analysis of this section in the Ramsey model. This would allow for investment in productive capacity, for production to depend on the capital stock, and for investment in a stock of clean technology, T, say $\alpha(T)$. The resource constraint for the economy would then be given by

$$\dot{K} = f(K) - C - \dot{T} - \delta(K + T). \tag{8.9}$$

This is left for further research.

9. Renewable and Exhaustible Environmental Resources

The models presented so far call for numerous extensions and generalisations. It is not the objective of the present paper to give an exhaustive account of all possible directions that future research can take. It seems, however, important to stress the interrelationship between the environment and renewable and/or exhaustible resources. This section provides an illustrative example of how this can be done. The stock of environmental resources, E, is depleted, because it is used up as a factor of production, R, (e.g., trees in the production of paper or other flora, fauna, coal, gas, etc.). This is detrimental to social welfare, both directly (less woods to enjoy) and indirectly as there are only limited resources, so eventually the current level of production and consumption may no longer be sustainable. The social planner maximises the social welfare function

$$W \equiv \int_0^\infty \exp(-\theta t)\,[B(C) - D_F(\alpha f(K)) - D_S(S) + U(E)]\,\mathrm{d}t \quad (9.1)$$

subject to

$$\dot{K} = f(K, R) - C - \delta K, \quad (9.2)$$

$$\dot{S} = \alpha f(K, R) - \sigma S \quad (9.3)$$

and

$$\dot{E} = H(E, S) - R, \quad (9.4)$$

where $U(E)$, $U' > 0$, $U'' < 0$ denotes the direct utility society derives from the stock of environmental resources. There are three dynamic equations: one for the physical capital stock, one for the stock of pollutants and one new one for the stock of environmental resources. The case of exhaustible resources corresponds to $H(E, S) = 0$. The general case of renewable resources allows for saturation in growth, $H_E > 0$ for $E < \bar{E}$, $H_E < 0$, for $E > \bar{E}$ and $H_{EE} < 0$, and allows pollution to have an adverse effect on the natural replenishment rate, $H_S < 0$. The properties of the special case of this problem which ignores the direct effects of the stock of environmental resources on social welfare is briefly discussed in Tahvonen and Kuuluvainen (1990).

As a first shot, it seems sensible to focus on the environmental externalities only, so it is assumed that $H(E, S) = H(E)$ and $D_F = D_S = 0$. The modified Keynes—Ramsey rule becomes

$$\dot{C} = [f_K(K, R) - \theta - \delta]\eta(C)C \quad (9.5)$$

whilst the social value of the environment to society, say e, must satisfy

$$\dot{e} = [\theta - H'(E)]e - U'(E) \tag{9.6}$$

and the use of the environmental factor of production follows from

$$B'(C)f_R(K, R) = e. \tag{9.7}$$

Equation (9.5) is familiar. Equation (9.6) says that the value of the stock of environmental resources to society is equal to the discounted value of future marginal utilities, where the social rate of discount is equal to the pure rate of time preference minus the natural replenishment rate of the stock of environmental resources. Equation (9.7) says that the marginal benefits of environmental resources in production should equal the marginal value of these resources to society.

If the production function displays constant returns to scale in capital and resources, the marginal products of the intensive-form production function are homogeneous of degree zero. Upon solution of equation (9.7) for R and substitution of the result into equations (9.2), (9.4) and (9.5), one obtains:

$$\dot{K} = Kf(1, \phi(E, C)) - C - \delta K \tag{9.2'}$$

$$\dot{E} = H(E) - K\phi(e, C), \quad \phi_e < 0, \quad \phi_C < 0 \tag{9.3'}$$

$$\dot{C} = [\psi(e, C) - \theta - \delta]\eta(C)C, \quad \psi_e < 0, \quad \psi_c < 0. \tag{9.4'}$$

The comparative statics of this model are left for further research.

10. Concluding Remarks

This paper addressed the problem of simultaneously determining the socially optimal stock of physical capital and environmental quality and examined how Pigouvian taxes can be used to enforce these in a decentralised economy. Both flow and stock externalities in the control of pollution were considered. These externalities arise, because pollution is a by-product of production and is mostly gradually dissolved in the environment. Optimal investment in clean technology and optimal abatement activities were also considered. When matters are left to the market, cleaning up of the environment does not take place and the levels of production and consumption will be too high from society's point of view. The government should step in and levy emission charges, clean up the environment and provide incentives to invest in clean technology. When one allows for pollution of the environment in the classical Ramsey problem, the capital stock is generally less than under the modified golden rule (which prevails under the market outcome) and a fortiori less than under the golden rule. As a consequence, the rate of consumption is also less. The problem the policy makers are facing is thus to reduce consumption and to direct investments to clean technology and abatement activities. This can be achieved by appropriate (dynamic) Pigouvian taxation. Of course, it is less appropriate in this long-run setting to rely on the Coase Theorem, as intergenerational bargaining opportunities are hard to establish.

For the models that have been used, it has been shown that there exists a steady state which is locally asymptotically stable and whose comparative statics properties have been analysed. The paper also reviewed and, whenever possible, provided a synthesis of environmental problems in the extended neoclassical Ramsey framework of optimal capital accumulation and economic growth. There remain many open questions. For example, the relationship between the models studied here and those models concerned with renewable and exhaustible resources deserves more attention. Future research should tackle the difficult problems involved in analysing such integrated models.

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Appendix

(a) Local stability analysis for Sections 6 and 7.
Define $\mu := -\tau$. μ is the (negative) shadow price of the stock of pollutants. Consider the system of equations (4.2), (4.3), (7.1)—(7.4). C can be solved from $B'(C) = \lambda$, to yield

$$C_\lambda = 1/B'', \quad C_\mu = 0, \quad C_S = 0.$$

Then A is solved from $-\mu\sigma'(A)S = \lambda$, giving

$$A_\lambda = \sigma'\lambda\sigma'', \quad A_\mu = \sigma'\sigma'S/\lambda\sigma'', \quad A_S = \sigma'\sigma'\mu/\lambda\sigma''.$$

Next (4.2), (4.3), (7.3) and (7.4) are linearized in the steady state. Straightforward calculations give the following Jacobian matrix,

$$J = \begin{pmatrix} -f' + \theta + \delta & -\alpha f' & -(\lambda + \alpha\mu)f'' & 0 \\ \mu\sigma'\sigma'/\lambda\sigma'' & (\sigma + \mu S\sigma'\sigma'\sigma'/\lambda\sigma'') + \theta & 0 & (\mu\mu\sigma'\sigma'\sigma'/\lambda\sigma'') + D'' \\ -1/B'' - \sigma'/\lambda\sigma'' & -S\sigma'\sigma'/\lambda\sigma'' & f' - \delta & -\mu\sigma'\sigma'/\lambda\sigma'' \\ -S\sigma'\sigma'/\lambda\sigma'' & -SS\sigma'\sigma'\sigma'/\lambda\sigma'' & \alpha f' & -\sigma - \mu S\sigma'\sigma'\sigma'/\lambda\sigma'' \end{pmatrix}$$

Or alternatively

$$J = \begin{pmatrix} H_{11} + \theta I & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

where for all i and j H_{ij} are 2×2 and $H_{11} = -H_{22}^T$, $H_{21} = H_{21}^T$, $H_{12} = H_{12}^T$.

Define

$$Q = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}.$$

Then it is easily verified that $QJQ^{-1} = -J^T + \theta I$. Let ψ be an eigenvalue of J and let x be the corresponding eigenvector. Then

$$QJx = Q\psi x = \psi Qx = -J^T Qx + \theta IQx.$$

So $-\psi + \theta$ is an eigenvalue as well.

(b) Comparative statics analysis for Section 7 ($\sigma' \neq 0$).
Upon substitution of λ and τ from (7.1) and (7.2) into (7.3) and (7.4) the following system results for the steady state.

$$((\alpha/\sigma'S)-1)f' + (\theta+\delta) = 0$$

$$((\theta+\sigma)B'/\sigma'S) - D'_S = 0$$

$$\alpha f - \sigma S = 0$$

$$f - \delta K - C - A = 0$$

where the arguments of the functions involved have been omitted. Straightforward but tedious calculations lead to the following system

$$\left\{\left(\frac{\alpha}{\sigma'S}-1\right)f'' - \frac{\alpha^2\sigma''Sf'f'}{(\sigma'S)^3}\right\}\mathrm{d}K + \left\{-\frac{\alpha\sigma'f'}{(\sigma'S)^2} + \frac{\alpha\sigma''f'S\sigma}{(\sigma'S)^3}\right\}\mathrm{d}S$$

$$+\left\{-\frac{\alpha\sigma''f'fS}{(\sigma'S)^3} + \frac{f'}{\sigma'S}\right\}\mathrm{d}\alpha + \mathrm{d}\theta + \mathrm{d}\delta = 0 \qquad \text{(A.1)}$$

$$\left\{\frac{(\theta+\sigma)B''\theta}{\sigma'S} + \left(\frac{\sigma'B'}{\sigma'S} + (\theta+\sigma)\frac{-B'\sigma''S}{(\sigma'S)^2}\right)\frac{\alpha f'}{\sigma'S}\right\}\mathrm{d}K$$

$$+\left\{-\frac{(\theta+\sigma)B'\sigma'}{(\sigma'S)^2} - D''_S + \frac{(\theta+\sigma)B''}{\sigma'S}\ \frac{\sigma}{\sigma'S} + \right.$$

$$\left. + \left(\frac{\sigma'B'}{\sigma'S} + (\theta+\sigma)\frac{-B'\sigma''S}{(\sigma'S)^2}\right)\frac{-\sigma}{\sigma'S}\right\}\mathrm{d}S$$

$$+\left\{\frac{-(\theta+\sigma)B''f}{(\sigma'S)^2} + \left(\frac{\sigma'B'}{\sigma'S} + (\theta+\sigma)\frac{-B'\sigma''S}{(\sigma'S)^2}\right)\frac{f}{\sigma'S}\right\}\mathrm{d}\alpha -$$

$$-\frac{(\theta+\sigma)B''K}{\sigma'S}\mathrm{d}\delta + \frac{B'}{\sigma'S}\mathrm{d}\theta = 0. \qquad \text{(A.2)}$$

Since $f'' < 0$, $\alpha/\sigma' S - 1 < 0$, $\sigma'' \leqslant 0$, $D'' > 0$ and $B'' < 0$ it is easily seen that all coefficients are positive except for the coefficients of $\mathrm{d}S$, which are negative, and the one corresponding with $\mathrm{d}K$ in (A.2) of which the sign is not uniquely determined. Therefore not much can be said about the comparative statics in the general case.

(c) Comparative statics analysis for Section 6 ($\sigma' = 0$).
For $\sigma' = 0$ the following system is obtained

$$\begin{pmatrix} (\alpha\tau - B')f'' & 0 & \alpha f' & -B''f' + (\theta + \delta)B'' \\ 0 & -D''_S & \theta + \sigma & 0 \\ f' - \delta & 0 & 0 & -1 \\ \alpha f' & -\sigma & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathrm{d}K \\ \mathrm{d}S \\ \mathrm{d}\tau \\ \mathrm{d}C \end{pmatrix} =$$

$$- \begin{pmatrix} \tau f' & B' & B' & 0 \\ 0 & \tau & 0 & \tau \\ 0 & 0 & -K & 0 \\ f & 0 & 0 & -S \end{pmatrix} \begin{pmatrix} \mathrm{d}\alpha \\ \mathrm{d}\theta \\ \mathrm{d}\delta \\ \mathrm{d}\sigma \end{pmatrix}.$$

The results given in the text originate from this system, where it should be realized that $D'' > 0$, $B'' < 0$, $f'' < 0$, $f' > \delta + \theta$, $\alpha\tau - B' < 0$.

Notes

[1] However, if the social damage function becomes, due to threshold effects, concave beyond a certain level of the stock of pollutants, two equilibria may exist, for example E and P. However, E is saddlepoint stable whilst P is globally unstable.
[2] In a utilitarian framework with population growth $\theta = \rho - n$, where ρ is the rate of pure time preference and n is the rate of population growth.
[3] Previous work employed models with two-sectors, one production sector and one abatement sector (Siebert, 1987; Musu, 1989).

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